MATHEMATICAL FUN & CHALLENGES IN THE GAME OF SET®

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The Game of SET

In 1988 Marsha Falco copyrighted a new game called SET. This game proves to be an excellent extension for activities involving organizing objects by attribute. In addition to reinforcing the ideas of sameness and distinctness, the SET game, and variations on it, provide an interesting and challenging context for exploring ideas in discrete mathematics. Even though the NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* includes discrete mathematics as a standard for grades 9-12, the activities suggested here are strongly supported by the K-4 and 5-8 standards involving mathematics as problem solving, communication, and reasoning.

The SET deck

The game of SET is a card game. A single card is identified by four attributes: number, shape, color, and shading. The full deck of cards form a complete set of all possible combinations of the four attributes. Each card has one, two or three (number) copies of the same figure showing. The figures are one of three shapes, colored with one of three colors, and shaded in one of three ways. In the commercial game, the shapes are

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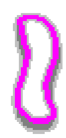
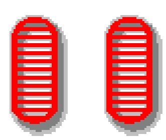


called "oval", "diamond" and "squiggle" respectively. Each of these shapes may be colored purple, red or green, and each is either outlined, filled in or striped. For example, the card in figure 1 has number 2, shape oval, color red, and shading striped. No two cards in the deck are identical and each possible choice of one value for each attribute occurs on one card.

Figure 1



When introducing SET in your classroom, challenge your students to describe the full deck of SET cards for themselves. Include in this challenge the question "Can you determine without counting the cards one by one, how many cards are in the complete SET deck?" Let the students have a deck to work with and ask them to figure out the rule by which the deck was constructed, or have the students construct a deck themselves and figure out in advance how many cards they will need. There are many ways children might arrive at the full count, usually involving some sorting of the cards.

The process of counting the SET deck cards without counting the cards one by one illustrates one the basic counting principles of discrete mathematics, called the multiplication principle. This principle says "if a first event can occur in n ways, and for each of these n ways a second event can occur in m ways, then the two events can occur in m x n ways. Here the "events" are the number of ways to assign attributes to the SET cards. For any card, one can choose 3 different number of figures to display, combined with one of three shapes for 9 combinations. Each of these 9 combinations can be paired with one of 3 colorings in 9 x 3 = 27 ways, each of which can be paired with 3 shadings for a total of 27 x 3 = 81 cards in the deck.

A 'Set' of Three
Sets of three cards from the SET deck which satisfy the condition that all the cards either agree with each other or disagree with each other on each of the four attributes (number, shape, color, and shading) are the fundamental objects in the SET game. Three cards form a 'set' if the cards display the same number of figures or each display a different number of figures, AND if the figures are all the same shape or three different shapes, AND if the figures are the same color or three different colors, AND if the figures are shaded with the same shading or three different shadings. For example, the cards in Figure 2 are a 'set', but the cards in Figure 3 are not a 'set'. Can you tell why?

Figure 2

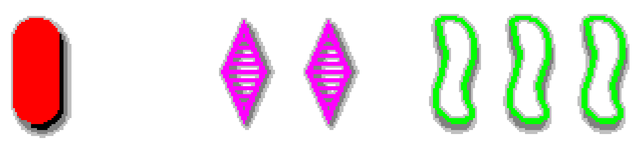
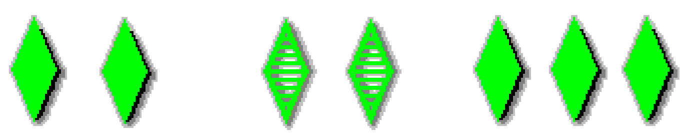


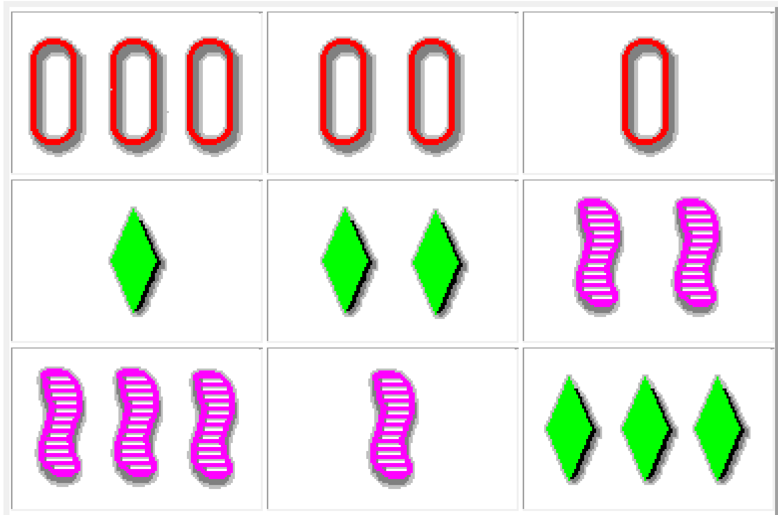
Figure 3



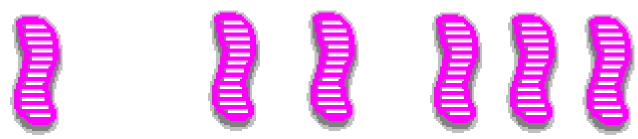
Playing SET
To begin the game of SET, the dealer shuffles the cards and lays some of them out in a rectangular array. (The official rules suggest beginning with 12 cards. From an educational point of view, it may be simpler for children to play beginning with 9 cards.) All players look at the same layout of cards seeking a 'set' of 3 cards as defined above. According to the official rules, there is a "MAGIC" rule: if two cards are.....and one is not ...., then it is not a 'set'."

To practice your understanding of the definition, see how many 'sets' you can find in Figure 4.

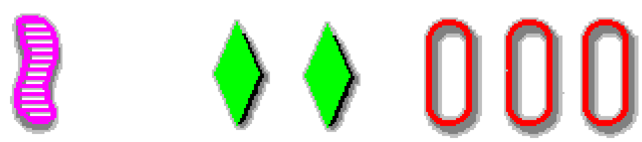
Figure 4



Did you find the 'set' consisting of:



How about the 'set' consisting of:

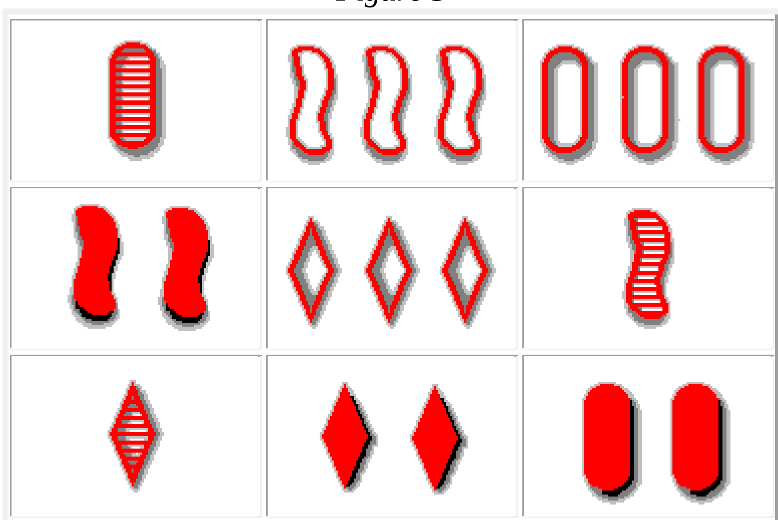


Notice that the three cards in a 'set' may be different in 1,2,3, or 4 of the attributes. The first person to notice a 'set' in the current layout calls out the word 'set' and then is allowed to touch the three cards. While it is not required in the rules, from a pedagogical point of view it is a good idea for the student to explain how s/he knows it is a 'set' -- for example the first 'set' above would be explained by saying, "they are all purple, all striped, all squiggles, and there is a 1, a 2, and a 3 of them." Assuming the student has correctly identified a 'set' s/he takes the 3 cards. If there are now fewer cards in the layout than at the start (i.e., 12 or 9), the dealer replaces them with three new cards. If all players agree there are no 'sets' in the layout, then 3 more cards are added. Play ends when no new cards are left in the deck and no 'sets' remain in the final array. The official game rules suggest that each player keep his/her own score by counting 1 point for each correctly identified 'set', and a -1 point for each incorrect attempt to identify a 'set'. The winner of the game is the player with the most points after each player has had a turn to deal the entire deck. When using SET in the classroom, we suggest a modification of the official rules. For beginners, don't exact any penalty for an incorrect attempt to identify a 'set'. Once students understand the game thoroughly, any student who makes an incorrect attempt may be penalized by not being allowed to call 'set" again until someone else has found a 'set'.

SET and Discrete Mathematics

As mentioned earlier, SET involves discrete math. According to John A. Dossey, "Discrete mathematics problems can be classified in three broad categories. The first category, *existence problems*, deals with whether a given problem has a solution or not. The second category, *counting problems*, investigates how many solutions may exist for problems with known solutions. A third category, *optimization problems*, focuses on finding a best solution to a particular problem."[1] The game of SET presents problems in both of the first two categories. One existence problem is to have each student pick out a random two cards from the SET deck and figure out how many, if any, cards can be found in the deck which can be paired with the first two cards to complete a set. It may take several selections of pairs of cards for students to realize that any pair can be completed to a 'set' by exactly one third card. Once students realize this, encourage them to explain to one another how they can be sure. The result holds for any pair of cards. A sample of such an argument might state: the unique third card is defined attribute by attribute -- for each attribute where the two chosen cards are alike, the third one has the same value; if they are different, the third one has the missing valve. Since only one card has each particular selection of four values for the four attributes, there is a unique completion for a 'set'. This activity supports an atmosphere of mathematics as communication and reasoning in your classroom. Those students who have had more experience counting combinations and permutations can be asked a more challenging question: if you pick any one card from the deck to how many distinct 'sets' does it belong? The answer requires the preceding result, namely that any two cards belong to exactly one 'set'. A particular card forms a 'set' with any of the 80 other cards in the deck with a unique third card to complete that 'set'. Each 'set' with the same beginning card is counted twice -- once with each of the other cards in the 'set' as the 'second' card selected. Thus, there are 80 /2 = 40 'sets' containing the first card.

Figure 5



Another question that junior high students might be able to answer is, "What is the largest number of 'sets' that can be present among a layout of nine cards?" A similar argument to the preceding one suggests that there are 9 possible first cards, each paired with 8 possible second cards -- but any of these cards in a particular 'set' can be 'first' and either of the remaining two can be "second" -- so there are (9x8)/(3x2) = 12 'sets' possible. The layout of Figure 5 is one example of nine cards (all of one color) including 12 'sets'. Can you find them all? Have your students construct their own examples of such layouts. See who can find a layout of 12 cards with the greatest number of 'sets'. Hint 14 is best possible.

Each of the suggested questions may be extended by varying the number of attributes or the number of options for attributes. What about a three-attribute deck with 5 possibilities for each attribute? There would be 125 cards in the deck, with a 'set' defined for a set of 5 cards.

There are many other games that can be played with the SET deck. The game and rules for variations can be obtained from Set Enterprises, Inc. 16537 E. Laser Dr., Ste. 6, Fountain Hills, AZ 85268. Other variations include the games that can be played with other sets of attribute blocks. For a book with many good ideas of attribute activities see [2].

As a final suggestion, 'set' is a word with meanings that are easily confused with the particular triples of the game SET. It might be better for children to call out some other word -- like 'triple' or three or '3-set' or some other word the class selects to describe the particular 'set' for this game.

Despite these minor concerns, the authors think the game of SET is a wonderful activity to add to the classroom -- it is thought provoking and fun!

References

[1] Dossey, John A., "Discrete Mathematics: The Math for our Time",
Discrete Mathematics Across the Curriculum K-12, 1991, NCTM, pp.1-2.

[2] **Teacher's Guide for Attribute Games and Problems**, Elementary Science Study,
Webster Division, McGraw-Hill Book Company, 1968, Educ. Dev. Ctr., Public Domain after 1971.

[3] **Curriculum and Evaluation Standards for School Mathematics**, NCTM, 1989.